

MEMORANDUM FOR THE RECORD

Declass Review by NGA.

SUBJECT: Measurement of X and Y Axis Perpendicularity

INTRODUCTION:

A machine used to make X and Y measurements in a plane can have, among others, an error due to non-perpendicularity of the two axes. There are several methods of testing for perpendicularity of the axes; such as optical tooling techniques, tracing of a calibrated grid, and application of the Pythagorean theorem to three measurements of a given length (on each axis and at a 40 to 50 degree angle). This note describes another simple method for checking perpendicularity using two-dimensional rulings.

It is necessary that tracking errors of the X and Y axes be small enough that the two-dimensional ruling can be accurately aligned with either axis. A necessary assumption is that the angular errors of the machine (α) and the two dimensional ruling (β) are small enough for $\tan(2\alpha) \approx (2\alpha) \approx \sin(2\alpha)$ and $\tan(2\beta) \approx (2\beta) \approx \sin(2\beta)$. It is obvious when referring to Figure 1 that just as $|\pi - 2\psi| = 2|\beta|$, we will have $|\pi - 2\theta| = 2|\alpha|$. The angle ψ is the angle between the positive y-axis and the positive x-axis of the machine being tested. The angle ψ is the angle between the two sets of accurate rulings on the two-dimensional ruling. The second ψ represents the essential second measurement where the test ruling has been rotated 90 degrees. The experimental procedure will yield, for the case of Figure 1, $(\alpha + \beta)$ and $(\beta - \alpha)$. The sum of these gives twice the angular error of the test ruling. Dividing their difference by two gives the machine's angular deviation from true perpendicularity.

The procedure to align one set of rulings of the test ruling with the X-axis of the machine is to scan with the machine and tap the test ruling iteratively until the scans show no significant deviations. This part of procedure will incidentally verify the straightness of both the test ruling and the machine. The desired measurement requires scanning a measured distance along the Y-axis of the machine and noting both the number of secondary lines (those not aligned with the X-axis) crossed by the machine's measuring mark and the direction of crossing. The testing ruling is then rotated 90 degrees in either direction and the procedure repeated. If the apparent direction of line crossings is the same in both cases, it shows that β is smaller than α in absolute value. A little thought will enable one to determine whether θ is acute or obtuse. Similar thinking will take care of the other cases. The number of secondary lines crossed can be converted to distance (knowing the ruling spacing) which divided by the Y-axis distance gives the angle between the Y-axis of the machine and the secondary rulings of the test ruling for the measurement. The Y-axis distance should be more than a few centimeters and can easily be measured independently of the machine so that Δx and Δy errors of the machine do not have to affect the accuracy of the measurement. Referring to Figure 1 again, the measured angles will be $|\alpha| \pm |\beta|$.

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RESULTS:*Ronchi no.
instead of line*

The left stages of two machines have been measured using a photographic rendering of a precision Ronchi ruling which was rotated after the first exposure and a second exposure made. The photographic emulsion was on a micro-flat glass plate and the ruling spacing was 666 lines per inch. The results for the first machine were 1221/406,386 and 348/400,000. The line crossings for the additive case $\{ ||\alpha| + |\beta| \}$ were to the left and for other case to the right, for Y increasing in all cases. This showed that θ was acute. Computational results are $|\beta| = 1.92$ milliradians (or 0.112 degrees) and $|\alpha| = 1.04$ milliradians (or 0.0596 degrees).

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The second machine had two measurements made with the same test ruling orientation and one measurement made after rotation. The angles were $\angle 1 = (6.5 \text{ rows})(25.4 \text{ mm/in.})/(666 \text{ rows/in.})(116.6 \text{ mm}) = 2.1$ milliradians, $\angle 2 = (7.0)(25.4)/(666)(804.7) = 2.2$ milliradians, and $\angle 3 = (6.9)(25.4)/(666)(120.3) = 2.2$ milliradians. The direction of line crossings for increasing Y were to the left for the first two measurements and to the right for the final measurement. Computation with angles $\angle 1$ and $\angle 3$ yields $|\beta| = 2.15$ milliradians and $|\alpha| = 0.05$ milliradians. Angle θ would be shown to be obtuse by the line crossing being to the right for the larger angle $\angle 3$ except for the fact that the experimental error is about twice the measured angle $|\beta| = 2.2$ milliradians and indicate that α is zero.

DISCUSSION:

The values of $|\beta|$ turned out to be 1.92, 2.15, and 2.2 milliradians for a maximum error of 14 per cent (or 0.28 milliradians). The error between the two sets of measurements of the second machine was about 2 per cent (or 0.05 milliradians). The source of error for these measurements are the pointing error of the machine's measuring mark, the uniformity of the two-dimensional test ruling, the length determination across the rulings, the length measurement along the Y-axis, and alignment of the test ruling with the X-axis of the machine. The test ruling used could be easily aligned with the X-axis to within one fourth of a line spacing over an X-axis distance comparable to the Y-axis travel used in the measurements. Since the readings involved about 6 lines, one fourth of a line gives an error of about 4 per cent. One operator has observed that his total pointing error on the first machine was less than 1 micron. Since $(25.4 \text{ mm/inch})(6 \text{ lines})/(666 \text{ lines/inch}) = 0.23 \text{ mm}$, this gives a pointing error of less than 0.43 per cent. Any length measurement along the Y-axis should be over several centimeters or more and be good to less than 1 per cent. The master Ronchi ruling used to make the two-dimensional test ruling has a straightness of 0.00005 in 10 inches and a parallelism of 0.00005 inches. With perfect X-axis alignment over 5 inches the angular error of the Y-axis from the center to either side would be one fourth of 0.00005 inches/10 inches or 0.0012 milliradians. This is negligible for the cases reported here. The parallelism error, in the worst case,

would introduce an error per line of 0.00005 inches. Since the spacing is 666 lines per inch the resulting error could be 3.33 per cent. This is comparable to the 4 per cent error noted previously for less than perfect alignment of the test ruling with the X-axis of the machine. Thus the measured 2 per cent error in α for the second machine is well within the present experimental error. Not enough is known about the very first measurement to pinpoint the source of the 14 per cent error with respect to the later measurements. However, a 14 per cent error for the corresponding α of 1.04 milliradians yields an angular tolerance of 1.04 ± 0.15 milliradians which is significantly different from zero.

In the foregoing procedure the angular error of interest is,

$$\frac{|\alpha| + |\beta| - ||\beta| - |\alpha||}{2} \quad \text{for} \quad |\alpha| < |\beta|$$

AND

$$\frac{|\alpha| + |\beta| + ||\beta| - |\alpha||}{2} \quad \text{for} \quad |\alpha| > |\beta|$$

Call this angle γ . To be certain that a machine is within the specified tolerance we must have $\gamma + \epsilon < \phi$ where ϕ is the specified angular tolerance and ϵ is the experimental error in the determination of γ . Obviously when $\gamma - \epsilon > \phi$ the machine is definitely out of tolerance. The machine is in the gray area when ϕ lies within $\gamma \pm \epsilon$. It may be noted that when no particular ϕ is specified and it has to be assumed equal to zero, there is no experimental test procedure that can produce any data that can establish with certitude that the machine is within tolerance. Of course it is possible to agree that if the gray area of $\gamma \pm \epsilon$ includes zero, ($\gamma < \epsilon$) the machine will be considered to be within tolerance. No known perpendicularity tolerance was assigned to the first machine. The natural assumption would be that it should have zero deviation. The tolerance for the second machine is said to be 5 seconds of arc which is 0.024 milliradians.

CONCLUSIONS:

The first machine at $|\alpha| = 1.04 \pm 0.15$ milliradians is definitely out of tolerance. The second machine at $|\alpha| = 0.05 \pm 0.10$ milliradians is within the gray area for a tolerance of 0.024 milliradians.


RECOMMENDATION:

Since there are several machines similar to the ones tested and since the second type of machine has limited refocussing range when it is in optical alignment, a larger, thinner, more accurate, and higher spatial frequency two-dimensional test ruling should be procured.

NOTE

A recent check showed that 0.050 inch thick spectroscopic plate (glass) changes the optical path length such that the focal setting is outside the allowed refocussing range of the second machine by more than five times its total permissible refocussing range. This means that the possibility of using especially stable photographic or other film materials should be considered.

Other machines in process should have provisions made such that a square glass plate with a test ruling on it can be substituted easily for the machine's platen.


Equipment Performance Branch, DED

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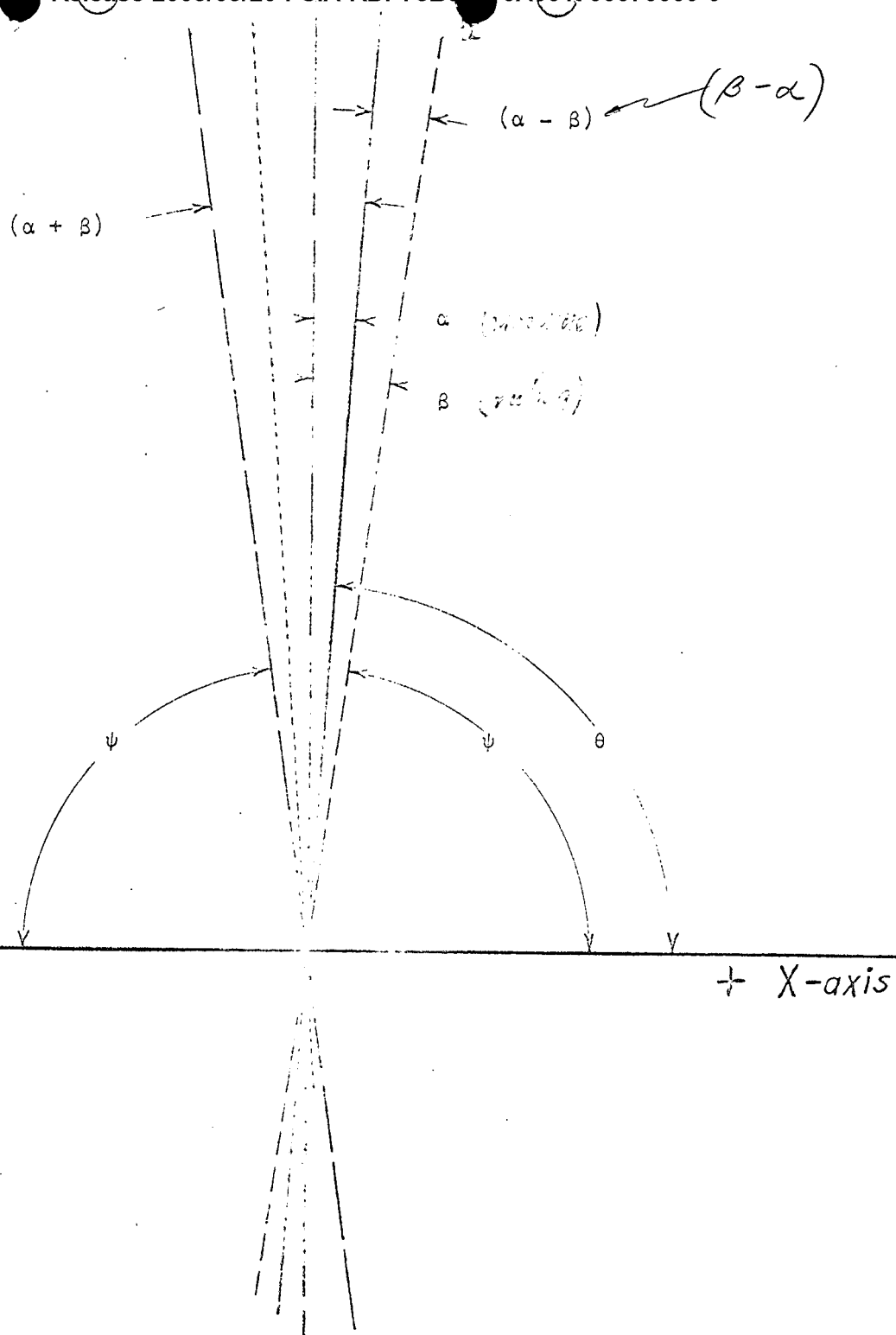


Figure 1
Angles for Perpendicularity Test